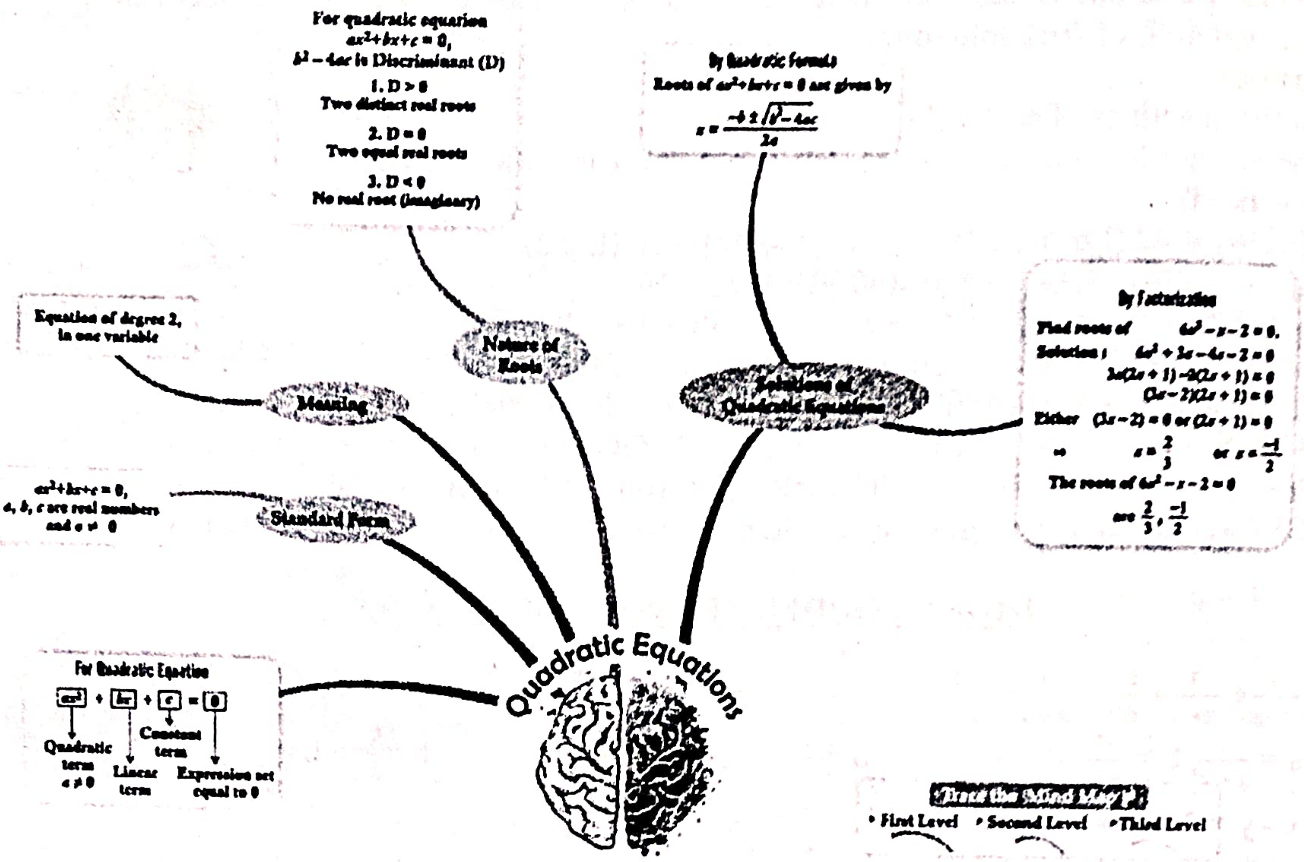
CHAPTER-4
QUADRATIC EQUATIONS
MIND MAPPING



GIST/SUMMARY OF THE CHAPTER

- Introduction
- Quadratic Equations: Standard form of a Quadratic Equation
- Solution of a Quadratic Equation by Factorisation Method, Quadratic Formula Method
- Nature of Roots

DEFINITIONS

QUADRATIC EQUATION

A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

The degree of a quadratic equation $ax^2 + bx + c = 0$ is 2,

STANDARD FORM:

The standard form (general form) of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

ROOTS:

A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$. (The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same)

The values of x that satisfy the quadratic equation, also called solutions.

METHODS OF SOLVING:

1.Factorization Method: Breaking down the quadratic equation into two linear factors.

2. Quadratic Formula: A direct formula for finding the roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, b^2 - 4ac \geq 0$$

NATURE OF THE ROOTS:

DISCRIMINANT (D):

The expression $b^2 - 4ac$, which determines the nature of the roots (distinct real roots, equal real roots, or complex roots).

- If $D = b^2-4ac > 0$: Two distinct real roots.
- If $D = b^2-4ac = 0$: Two equal real roots (repeated roots).
- If $D = b^2-4ac < 0$: No real roots.

Applications:

- Quadratic equations are used in various fields like Physics, Engineering, and Economics to model different scenarios.
- They can be used to solve problems involving Geometry, Projectile motion, and Optimization.

Method to Solve Word Problems:

Step-1 Firstly, Consider the first quantity as a variable (say x) and then find the other quantities in terms of x by using the given condition.

Step-2 According to the given condition, write the equation in the quantities which is obtained from step-1.

Step-3 Simplify the equation obtained in step-2 to get quadratic equation.

Step-4 Now, simplify the quadratic equation by any one of the methods (i.e. factorization and quadratic formula) to obtain the value(s) of variable x.

Step-5 Further check whether the value(s) of variable x, satisfies the given condition or not.

Step-6 Find the values of the quantities. Also, Find the values of other information, if asked in the question.

FORMULAE

1. **Solution of Quadratic Equation by Quadratic Formula**

$X = \frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$ or $X = \frac{-b\pm\sqrt{D}}{2a}$ where $D=b^2-4ac$ is known as Discriminant

This Formula is known as **Quadratic formula or Sridharacharya Formula**

2. Condition for two equal real roots $D=0$ i.e. $b^2-4ac=0$.

If $D=b^2-4ac=0$ then $x = \frac{-b}{2a}$, $x = \frac{-b}{2a}$

3. Condition for two distinct real roots $D > 0$ i.e. $b^2-4ac0 > 0$

If $D=b^2-4ac>0$ then

$X = \frac{-b+\sqrt{D}}{2a}$, $X = \frac{-b-\sqrt{D}}{2a}$

4. Condition for no real roots $D < 0$ i.e. $b^2-4ac<0$

If $D=b^2-4ac<0$ $\sqrt{D}$ can not be evaluated as there is no real roots.

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1. If the quadratic equation $3x+\frac{1}{x} = x+3$ is written in standard form, then

(a) a=3, b=3, c=1 (b) a=3, b=-3, c=1

(c) a=2, b=-3, c=1 (d) a=3, b=-3, c=3

Ans. (c) a=2, b=-3, c=1

$3x+\frac{1}{x} = x+3$, Multiplying throughout the equation by x

$3x^2+1=x^2+3 \Rightarrow 2x^2-3x+1$

2. For the equation $x^2 + 5x - 1$, which of the following statements is correct?

(a) The roots of the equation are equal (b) The discriminant of the equation is negative

(c) The roots of the equation are real, distinct and irrational (d) The discriminant is equal to zero

Ans. (c) The roots of the equation are real, distinct and irrational

$b^2-4ac = 5^2 - 4(1)(-1)$

$25 + 4 = 29$ which is greater than 0 and irrational.

3. If the roots of the equation $ax^2 + bx + c$ are real and equal, what will be the relation between a, b, c?

(a) $b^2 = 4ac$ (b) $b^2 = ac$ (c) $b^2 = 2ac$ (d) $b^2 = \sqrt{ac}$

Ans. (a) $b^2 = 4ac$

4. If $\frac{1}{2}$ is a root of the quadratic equation $x^2 - mx - \frac{5}{4} = 0$, then value of m is:

(a) 2 (b) -2 (c) -3 (d) 3

Ans. (b) -2, Given $x = \frac{1}{2}$ as root of equation $x^2 - mx - \frac{5}{4} = 0$.

$$\left(\frac{1}{2}\right)^2 - m\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \Rightarrow \frac{1}{4} - \frac{m}{2} - \frac{5}{4} = 0 \Rightarrow m = -2$$

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, the other two sides of the triangle are equal to:

(a) Base=10cm and Altitude=5cm (b) Base=12cm and Altitude=5cm
(c) Base=14cm and Altitude=10cm (d) Base=12cm and Altitude=10cm

Ans. (b) Base=12cm and Altitude=5cm

Let the base be x cm., Altitude = $(x - 7)$ cm

In a right triangle, $Base^2 + Altitude^2 = Hypotenuse^2$ (From Pythagoras theorem)

$\therefore x^2 + (x-7)^2 = 13^2$ By solving the above equation, we get;

$\Rightarrow x = 12$ or $x = -5$ Since the side of the triangle cannot be negative.

Therefore, base = 12cm and altitude = 12-7 = 5cm

6. The sum of a number and its reciprocal is $\frac{65}{8}$ Then the number is:

(a) $8, \frac{1}{8}$ (b) 4 (c) 2 (d) 6

Ans. (a) $x = 8, \frac{1}{8}$

Let the number be x then as per condition $x + \frac{1}{x} = \frac{65}{8}$

$8x^2 - 65x + 8 = 0$, $8x(x-8) - 1(x-8)$ so $x = 8, \frac{1}{8}$

7. If one root of equation $4x^2 - 2x + k - 4 = 0$ is reciprocal of the other. The value of k is:

(a) -8 (b) 8 (c) -4 (d) 4

Ans. (b) 8

If one root is reciprocal of others, then the product of roots will be:

$\alpha \times \frac{1}{\alpha} = \frac{k-4}{4} = 1 \Rightarrow k - 4 = 4$ so $k = 8$

8. The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 \ldots\ldots\ldots}}}$ is

(a) 4 (b) 3 (c) 3.5 (d) -3

Ans. (b) 3

Hint: we can write, $\sqrt{(6+x)} = x$

$x^2 - x - 6 = 0 \Rightarrow x^2 - 3x + 2x - 6 = 0 \Rightarrow x(x-3) + 2(x-3) = 0 \Rightarrow (x+2)(x-3) = 0 \Rightarrow x = -2, 3$

9. Which of the following is not a quadratic equation

(a) $x^2 + 3x - 5 = 0$ (b) $x^2 + x^3 + 2 = 0$ (c) $3 + x + x^2 = 0$ (d) $x^2 - 9 = 0$

Ans. (b) $x^2 + x^3 + 2 = 0$: Since it has degree 3.

10. The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is

(a) $\pm\sqrt{6}$ (b) ± 4 (c) $\pm 3\sqrt{2}$ (d) $\pm 2\sqrt{6}$

Ans. (d) $\pm 2\sqrt{6}$

Condition for two equal roots is $b^2 - 4ac = 0$

$\Rightarrow (k)^2 - 4 \times 2 \times 3 = 0 \Rightarrow k^2 = 24 \Rightarrow k = \pm\sqrt{24} \therefore k = \pm 2\sqrt{6}$

ASSERTION AND REASONING QUESTIONS

Directions: In the questions below, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct option.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

1. **Assertion(A):** $3x^2 - 6x + 3 = 0$ has repeated roots.
   Reason(R): The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.
   Ans. (c)
2. **Assertion(A):** If one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3, then the value of k is 2.
   Reason(R): The quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has almost two roots.
   Ans. (b)
3. **Assertion(A):** $(2x - 1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.
   Reason(R): An equation of the form $ax^2 + bx + c = 0, a \neq 0$, where a, b, c $\in$ R is called a quadratic equation.
   Ans. (a)
4. **Assertion(A):** The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary
   Reason(R): If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.
   Ans. (a)
5. **Assertion (A):** The quadratic equation $x^2+4x+5=0$ has no real roots.
   Reason (R): The discriminant is greater than zero.
   Ans. (c)
6. **Assertion (A):** The solution of the quadratic equation $(x-1)^2 - 5(x-1) - 6 = 0$ is 7
   Reason (R): The solution of the equation $x^2+5x-(a^2+a-6) = 0$ is a+3
   Ans. (c)
7. **Assertion (A):** Every quadratic equation has exactly one root.
   Reason(R): Every quadratic equation has at most two roots.
   Ans. (d)
8. **Assertion(A):** If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
   Reason (R): Every quadratic equation has at least two roots.
   Ans. (c)
9. **Assertion(A):** The equation $x^2 + 4x + 5 = 0$ has two equal roots.
   Reason (R): If discriminant $D = b^2 - 4ac = 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are equal.
   Ans. (a)
10. **Assertion(A):** The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary
    Reason(R): If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.
    Ans. (a)

VERY SHORT ANSWER TYPE QUESTIONS (2MARKS QUESTIONS)

1. Find the value of p so that the quadratic equation $px(x - 3) + 9 = 0$ has two equal roots.
   Ans: $px^2 - 3px + 9 = 0$, condition for equal roots: $b^2 - 4ac = 0 \Rightarrow (-3p)^2 - 4(p)(9) = 0$
   on solving we get $p = 0$ (rejected) or $p = 4 \therefore p = 4$($\because$ Coeff. of x^2 cannot be zero).
2. Find the roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant.

Ans: $D = b^2 - 4ac = (2m+3)^2$

$$x = \frac{-(-3) \pm \sqrt{(2m+3)^2}}{2(1)} = \frac{3 \pm (2m+3)}{2}$$

$$= \frac{3+2m+3}{2} \text{ or } \frac{3-2m-3}{2}$$

$$x = \frac{2(m+3)}{2} = \frac{-2m}{2}$$

$\therefore x = m+3$ or $-m$

3. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then find the value of ab.
   Ans: On putting the value of y in $ay^2 + ay + 3 = 0$, we get $a = -\frac{3}{2}$
   Again, putting the value of y in $y^2 + y + b = 0$, we get $b = -2$ so $ab = 3$.
4. Check if $x(x+1) + 8 = (x+2)(x-2)$ is in the form of quadratic equation.
   Ans: $x(x+1) + 8 = (x+2)(x-2) \Rightarrow x^2 + x + 8 = x^2 - 2^2$. On simplification we get $x + 12 = 0$.
   Since, this expression is not in the form of ax^2+bx+c, hence it is not a quadratic equation.
5. Find two consecutive positive integers, the sum of whose squares is 365.
   Ans: Let us say, the two consecutive positive integers be x and $x+1$.
   $x^2 + (x+1)^2 = 365 \Rightarrow x^2 + x - 182 = 0 \Rightarrow x = -14$ or $x = 13$
   $x + 1 = 13 + 1 = 14$. so, the two consecutive positive integers will be 13 and 14.
6. Is it possible to design a rectangular park of perimeter 80 and area 400 sq.m.? If so find its length and breadth.
   Ans: Let the length and breadth of the park be L and B.
   Perimeter of the rectangular park $= 2(L+B) = 80$
   $\Rightarrow$ Area of the rectangular park $= L \times B \Rightarrow L^2 - 40L + 400 = 0$,
   on solving the equations we get L=20m and B=20m.
7. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.
   Ans: Given that $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$
   $\Rightarrow (3)^2 - 2k(3) - 6 = 0$
   $\Rightarrow k = ½$
8. Find the quadratic polynomial if its zeroes are 0, $\sqrt{5}$.
   Ans. A quadratic polynomial can be written using the sum and product of its zeroes as:
   $x^2 - (\alpha+\beta)x + \alpha\beta$, where α and β are the roots of the polynomial. Here, $\alpha = 0$ and $\beta = \sqrt{5}$,
   $\Rightarrow x^2 - (0+\sqrt{5})x + 0(\sqrt{5}) \Rightarrow x^2 - \sqrt{5}x$.
9. Find the value of "x" in the polynomial $2a^2 + 2xa + 5a + 10$ if $(a+x)$ is one of its factors.
   Ans. Let $f(a) = 2a^2 + 2xa + 5a + 10$, Since, $(a+x)$ is a factor of $2a^2 + 2xa + 5a + 10$, $f(-x) = 0$
   So, $f(-x) = 2x^2 - 2x^2 - 5x + 10 = 0$ Therefore, $x = 2$.
10. How many zeros does the polynomial $(x-3)^2 - 4$ have? Also, find its zeroes.
    Ans. On expanding the given expression, we find $x^2 - 6x + 5$. As the polynomial has a degree of 2, the number of zeroes will be 2. On solving $x^2 - 6x + 5 = 0$ we get the roots $x = 1, x = 5$.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.
   Ans. $3x^2 - 2x + \frac{1}{3} = 0$, Since, Discriminant $= b^2 - 4ac \Rightarrow (-2)2 - 4 \times 3 \times \frac{1}{3} \Rightarrow 0$
   Hence, the given quadratic equation has two equal real roots. The roots are $\frac{-b}{2a}$ and $\frac{-b}{2a}$.
   on solving the roots are $\frac{1}{3}, \frac{1}{3}$
2. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.
   Ans. $px^2 - 14x + 8 = 0$, Let α and 6α be the roots of the given quadratic equation.
   Sum of the roots = −coefficient of x/coefficient of $x^2 \Rightarrow \alpha + 6\alpha = -(-14)/p$ ...

Product of roots = constant term/coefficient of $x^2 \Rightarrow (\alpha)(6\alpha) = \frac{8}{p}$ on substituting $\alpha = \frac{2}{p}$ from (i), we get p = 3.

3. Solve for x: $\sqrt{2x+9} + x = 13$.
   Ans. $\sqrt{2x+9} + x = 13$, on squaring both the sides, $x^2 - 28x + 160 = 0 \Rightarrow x = 8$ or $x = 20$
4. Solve for x: $4x^2 - 4ax + (a^2 - b^2) = 0$
   Ans. $4x^2 - 4ax + (a^2 - b^2) = 0$
   $\Rightarrow [4x^2 - 4ax + a^2] - b^2 = 0 \Rightarrow (2x - a)^2 - (b)^2 = 0 \Rightarrow (2x - a + b)(2x - a - b) = 0$
   $\Rightarrow 2x - a + b = 0$ or $2x - a - b = 0 \Rightarrow 2x = a - b$ or $2x = a + b \therefore x = (a-b)/2$ or $x = (a+b)/2$
5. Find that value of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0, p \neq -1$ had equal roots.
   Ans. For the given quadratic equation to have equal roots, D = 0
   Here $a = (p+1)$, $b = -6(p+1)$, $c = 3(p+9)$
   $D = b^2 - 4ac$
   $\Rightarrow [-6(p+1)]^2 - 4(p+1).3(p+9) = 0 \Rightarrow 36(p+1)^2 - 12(p+1)(p+9) = 0$
   $\Rightarrow p + 1 = 0$ or $p - 3 = 0 \Rightarrow p = -1$ (rejected) or $p = 3 \therefore p = 3$.

6. α and β are zeroes of the quadratic polynomial $x^2 - 6x + y$. Find the value of 'y' if $3\alpha + 2\beta = 20$.
   Ans. Let, $f(x) = x^2 - 6x + y$ from the given $3\alpha + 2\beta = 20$—(i), $\alpha + \beta = 6$—(ii) and, $\alpha\beta = y$—(iii) on solving $\alpha = 8$, $\beta = -2$. $y = \alpha\beta = (8)(-2) = -16$.
7. Find two numbers whose sum is 27 and product is 182.
   Ans. Let us say the first number is x, and the second number is 27 – x. As per question: $x(27 - x) = 182 \Rightarrow x^2 - 27x - 182 = 0$, on solving x = 13 or x = 14 the numbers are 13 and 14
8. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is 1/3. Find his present age.
   Ans. Let the present age of Rahman is x years. Three years ago, Rehman's age was $(x - 3)$ years. Five years after, his age will be $(x + 5)$ years. As per the condition given
   $\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$ after simplification we get: $x^2 - 4x - 21 = 0 \Rightarrow x = 7, -3$
   As we know, age cannot be negative. Therefore, Rahman's present age is 7 years
9. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.
   Ans . Let us say the marks of Shefali in Math be x. Then, the marks in English will be 30 – x.
   As per the given question, $(x + 2)(30 - x - 3) = 210$, on solving the QE we get x=12,13
   Therefore, if the marks in Math are 12, then marks in English will be 30 – 12 = 18, and if the marks in Math are 13, then marks in English will be 30 – 13 = 17.
10. Sum of the areas of two squares is 468 m^2. If the difference between their perimeters is 24 m, find the sides of the two squares.
    Ans. Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$, respectively and the area of the squares will be x^2 and y^2 respectively.
    As per question $4x - 4y = 24$, Also, $x^2 + y^2 = 468$.
    By these two equations we obtain QE: $y^2 + 6y - 216 = 0$. After simplification we get $y = -18, 12$.
    As we know, the side of a square cannot be negative.
    Hence, the sides of the squares are 12 m and (12 + 6) =18 m.

LONG ANSWER TYPE QUESTIOS (5 MARKS QUESTIONS)

1. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the original duration of the flight.
   Ans. Let the speed of the flight be x km/hrs. According to the given, $\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$
   After simplification we get $x^2 - 200x - 2400 = 0$, on solving this QE we obtain x = -400, x = 600. Time cannot be negative. Therefore, x = 600 km/hrs.

Hence, the original duration of the flight is 1 hr.

2. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Ans. Let the base of the right triangle is x cm, the altitude of right triangle = $(x-7)$ cm
By Pythagoras' theorem, $x^2+(x-7)^2=13^2$ on solving we get $x=12$ or $x=-5$
Since sides cannot be negative, x can only be 12. Therefore, the base of the given triangle is 12 cm, and the altitude of this triangle will be $(12-7)$ cm = 5 cm.

3. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Ans. Let the number of articles produced is x.
Therefore, cost of production of each article = Rs $(2x+3)$. Given the total cost of production is Rs.90
$\therefore x(2x+3)=90$ on solving QE we get $x=-15/2$ or $x=6$. As the number of articles produced can only be a positive integer, x can only be 6. Hence, the number of articles produced = 6
so the Cost of each article = $2\times 6+3$ = Rs 15.

4. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Ans. Let us say the average speed of the passenger train = x km/h.
Average speed of express train = $(x+11)$ km/h. As per question: $\frac{132}{x}-\frac{132}{x+11}=1$
On solving Q E $x^2+11x-1452=0 \Rightarrow x=-44, 33$. As we know, speed cannot be negative. Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be $33+11=44$ km/h.

5. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by 1/15. Find the fraction.

Ans. Let the denominator be x and the numerator be $x-3 \therefore$ Fraction $=\frac{x-3}{x}$
New denominator = $x+1$
According to the Question,

$$\Rightarrow \frac{x-3}{x+1}=\frac{x-3}{x}-\frac{1}{15}$$

$$\Rightarrow \frac{x-3}{x+1}=\frac{15x-45-x}{15x}$$

$$\Rightarrow \frac{x-3}{x+1}=\frac{14x-45}{15x}$$

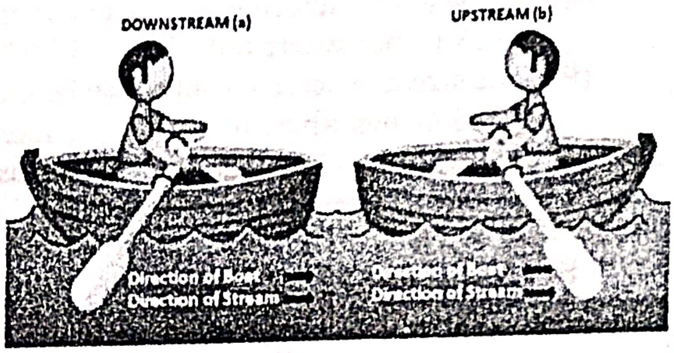
after simplification we get $\Rightarrow x^2-14x+45=0 \Rightarrow x=5$ or $x=9$
When $x=5$, fraction $=\frac{2}{5}$ When $x=9$, fraction $=\frac{2}{3}$ $\therefore$ Fraction $=\frac{2}{5}$ or $\frac{2}{3}$

CASE BASED QUESTIOS (4 MARKS QUESTIONS)

1. Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.

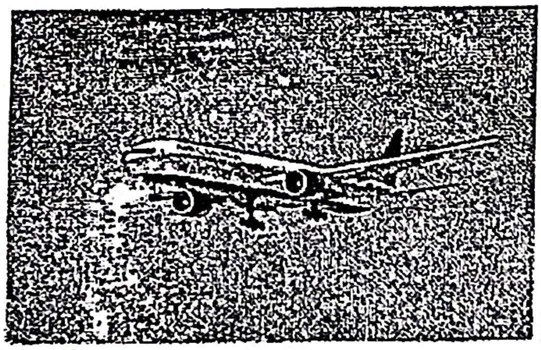
(i) What will be the distance covered by Ajay's car in two hours?
(ii) Write a quadratic equation which describe the speed of Raj's car?
(iii) What is the speed of Raj's car?
(iv) How much time took Ajay to travel 400 km?
Ans. (I) $2(x + 5)$km, (ii) $x^2 + 5x - 500 = 0$, (iii) 20km/h, (iv) 16 hours

2. The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.



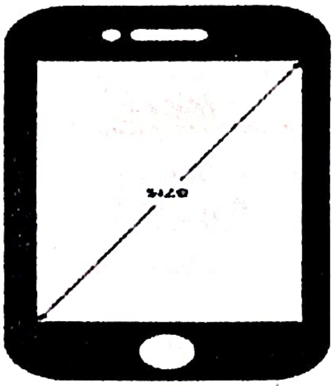
(i)Let speed of the stream be x km/hr. then what will be the speed of the motorboat in upstream.
(ii)Write the correct quadratic equation for the speed of the current?
(iii)How much time boat took in downstream?
(iv)What is the speed of the current?
Ans. (i) (20-x) km/h, (ii) $x^2 + 30x - 400 = 0$, (iii) 45 minutes, (iv) 10km/h

3. An aero plane travelled a distance of 400km at an average speed of x km/hr. On the return journey, the speed was increased by 40km/hr.



(i)Write an expression for times taken for the onward journey.
(ii)Write an expression for time taken for the return journey.
(iii)If the return journey took 30 minutes less than the onward journey, then the what is the equation formed in x.
(iv)Find the value of equation obtained in question (iii)
Ans. (i) $\frac{400}{x}$ (ii) $\frac{400}{x+40}$ (iii) $x^2 + 40x - 32000 = 0$, (iv) 160

4. Digital images consist of pixels. A pixel can be considered as the smallest unit on a display screen in a mobile or a computer. The number of pixels, their size and colours depend on the display screen and its graphic card. Display screens are rectangular in shape and their size is defined as the length of the diagonal. Amit is designing a web page for a display on a screen whose size is 1000 pixels. The width of the screen is 800 pixels.



(i) Write a quadratic equation which is used to calculate the height (h) of the screen?

(ii) The size of a screen display can be measured in inches also. Is it possible to have a screen of size 13 inches where the width is 7 inches more than height? Give an example to justify.

(iii) The size of a screen display can be measured in inches also. Is it possible to have a screen of size 13 inches where the width is 7 inches more than height? Give an example to justify.

Ans: (i) $h^2 - 200 \times 1800 = 0$, (ii)Yes, uses Pythagoras' theorem (iii) $w^2 - 430w + 15{,}600$

5. Two students Preet and Vihar were solving a particular problem. The problem is as follows. Some students (say 'x') of Kendriya Vidyalaya, planned a picnic during autumn break. The budget for f food was Rs. 480. But eight of these failed to goand, thus, the cost of food for each member increased by Rs. 10.

Based on the above information answer the following.

(i) If all students are going to the picnic, then, what is the cost of food for each student?

(ii) If 8 students to fail to go then, what is the cost of food per student?

(iii)Write the equation to find the value of 'x'.

(iv)Solve the equation: $x^2-8x-384=0$

Ans. (i) $\frac{480}{x}$ (ii) $\frac{480}{x-8}$ (iii) $x^2-8x-384=0$ (iv) x= 24 and x= -16.

HIGHER ORDER THINKING SKILL QUESTIONS

1. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.

Ans. Let one pipe fills the cistern in x mins. Therefore, the other pipe will fill the cistern in (x+3) mins. Time taken by both, running together, to fill the cistern = $3\frac{1}{13}$ mins= $\frac{40}{13}$mins

Part filled by one pipe in 1 min == $\frac{1}{x}$ Part filled by the other pipe in 1 min = $\frac{1}{x+3}$

Part filled by both pipes, running together, in 1 min = $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$.

after simplification we get $13x^2 - 41x - 120 = 0$

On solving above QE we get $x=5$ or $\frac{-24}{13}$. Thus, one pipe will take 5 mins and other will take $\{(5+3)=8\}$ mins to fill the cistern.

2. Solve for x: $9x^2-9(a+b)x+(2a^2+5ab+2b^2)=0$

   Ans. $9x^2-9(a+b)x+(2a^2+5ab+2b^2)=0$.

   The above equation can be rewritten as $9x^2-9(a+b)x+(a+2b)(2a+b)=0$.

   On solving the above equation, we obtain $x=\frac{a+2b}{3}$ or $\frac{2a+b}{3}$.

3. A lotus is 2m above the water in a pond due to wind the lotus slides on the side and only the stem completely summers in the water at a distance of 10m from the original position find the depth of the water in the pond.

   Ans. Let the depth of the pond be x m. According to question $(x+2)^2= x^2+10^2$.

   On solving the above QE we get x=24 m. Depth of the pond is 24 m.

4. A teacher on attempting to arrange the students for mass drill in the form of a solid square found that 24 students were leftover. When he increased the size of the square by 1 student he found he was short of 25 students. Find the number of students.

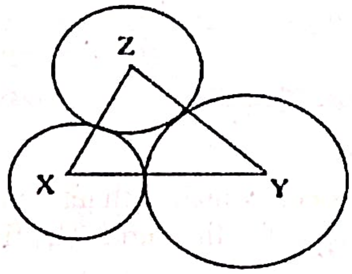
   Ans. Let the side of the square be x m. No. of students = x^2+24, New side $=x+1$,.

   so the number of students = $(x+1)^2-25$.

   ATQ $x^2+24= (x+1)^2-25$.

   On solving we get x=24. Number of students = 576+24= 600.

5. X and Y are the centers of circles of radius 9 cm and 2 cm and XY =17 cm. Z is the center of a circle of radius 4 cm which touches the above circle externally. Given that $\angle XZY = 90^0$ write an equation in r and solve it for r.



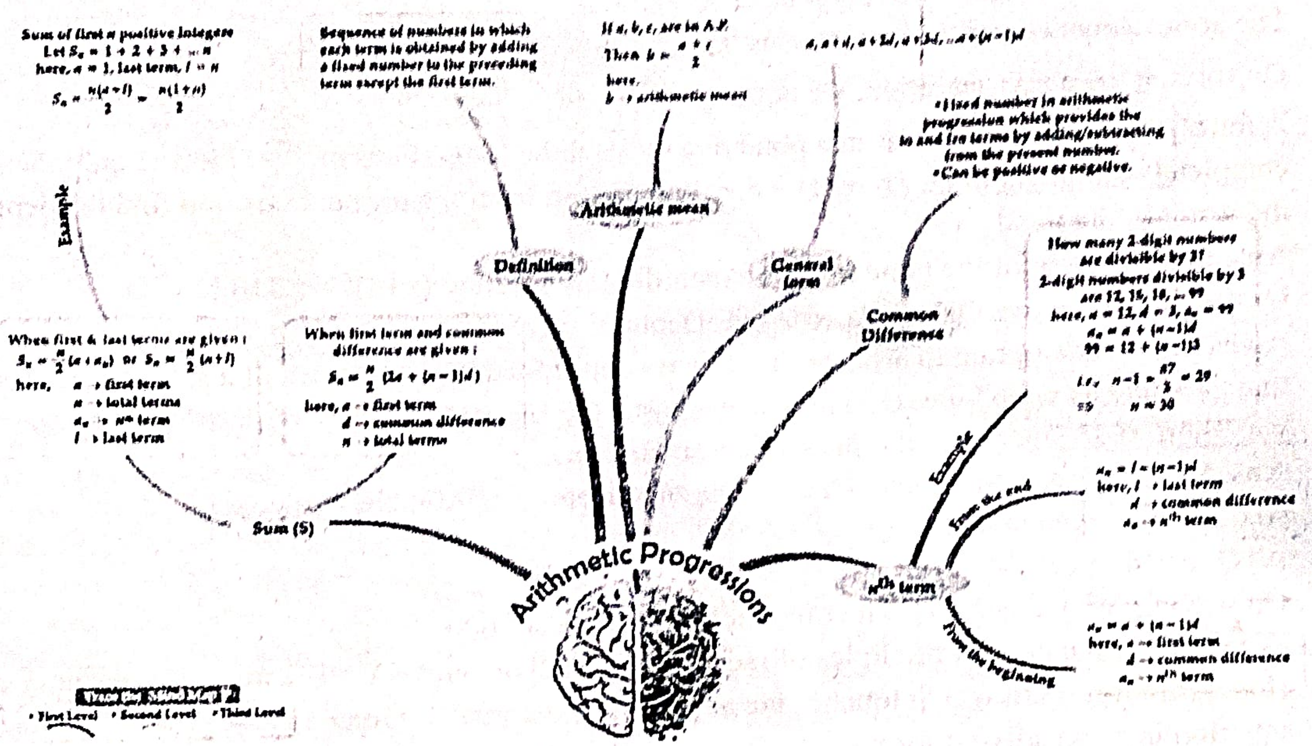
Ans. Let r be the radius of third circle. XY=17cm

$\Rightarrow XZ=9+r$, $YZ=r+2$.

ATQ $(r+9)^2+(r+2)^2=17^2$.

After simplification we get $r^2+11r-102=0$, on solving r=6 cm or r = -17(not possible), so radius= 6cm.

CHAPTER-5
ARITHMETIC PROGRESSIONS
MIND MAPPING:



GIST OF THE CHAPTER

- Introduction
- Arithmetic Progressions
- nth Term of an AP
- Sum of First n Terms of an AP

DEFINITIONS

1.An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference.

The general form of an AP is a, a + d, a + 2d, a + 3d, . . .

2. A given list of numbers a_1, a_2 , a_3, . . . is an AP, if the differences $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, . . ., give the same value, i.e., if ak + 1 – a k is the same for different values of k.

3. In an AP with first term a and common difference d, the nth term (or the general term) is given by $a_n = a + (n - 1) d$.

4. The sum of the first n terms of an AP is given by:

$$S = \frac{n}{2}[2a+(n-1)d]$$

5. If l is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by: $S = \frac{n}{2}[a+l]$

FORMULAE

- Arithmetic Sequence in general form: a, a + d, a + 2d, a + 3d, . . .
- AP Formula to find common difference: $d = a_2 - a_1$
- Arithmetic progression formula for nth term: $a_n = a + (n - 1) d$, where 'a' depicts the constant term, 'n' is the number of terms and 'd' is the common difference of the AP.
- The sum of first 'n' terms of arithmetic progression when the nth term is NOT known is: $S = \frac{n}{2}[2a+(n-1) d]$

- The sum of first n terms of an arithmetic progression when the nth term, an is known: is $S_n = \frac{n}{2}[a_1 + a_n]$
- The nth term from the last in an Arithmetic Progression (AP) , $a_n = l - (n - 1) d$ where 'l' is the last term of the AP, 'n' is the term number from the last, and 'd' is the common difference.
- The formula for finding the nth term (a_n) of an arithmetic progression (AP) when the sum of the first n terms (S_n) is given, is: $a_n = S_n - S_{n-1}$.
  This formula essentially states that the nth term is equal to the difference between the sum of the first n terms and the sum of the first (n-1) terms.
- If the terms are represented as a, b, and c, then the relationship is 2b = a + c. In simpler terms, the middle term is the average of the first and third terms.
- If the sum of three terms of an arithmetic progression (AP) is given, then the terms are
  a - d, a, and a + d, where 'a' is the first term and 'd' is the common difference.
- If the sum of four terms of an arithmetic progression (AP) is given, then the terms are
  a - 3d, a - d, a + d, and a + 3d, where 'a' is the first term and 'd' is the common difference.

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1. The n^{th} term of an A.P. is given by $a_n = 3 + 4n$. The common difference is
   (a) 7 (b) 3 (c) 4 (d) 1
   Ans: (c) 4 [$d = a_2 - a_1$, 11-7=4]
2. If p, q, r and s are in A.P. then r – q is
   (a) s – p (b) s – q (c) s – r (d) none of these
   Ans: (c) s – r
3. If the sum of three numbers in an A.P. is 9 and their product is 24, then numbers are
   (a) 2, 4, 6 (b) 1, 5, 3 (c) 2, 8, 4 (d) 2, 3, 4
   Ans: (d) 2, 3, 4[By using three terms a-d, a, a+d]
4. The $(n-1)^{th}$ term of an A.P. given by 7,12,17, 22,... is
   (a) 5n + 2 (b) 5n + 3 (c) 5n – 5 (d) 5n – 3
   Ans: (d) 5n – 3 [$a_{n-1} = a + [(n-1) - 1]d = 7 + [(n-1) - 1](5) =$
   $7 + (n-2)5 = 7 + 5n - 10 = 5n - 3$]
5. The 10^{th} term from the end of the A.P. -5, -10, -15,..., -1000 is
   (a) -955 (b) -945 (c) -950 (d) -965
   Ans: (a) -955 [10^{th} term from the end $= l - (n-1)d = -1000 - (10-1)(-5) = -1000 + 45 = -955$]
6. Find the sum of 12 terms of an A.P. whose nth term is given by $a_n = 3n + 4$
   (a) 262 (b) 272 (c) 282 (d) 292
   Ans: (c) 282 [$a_1 = 7$, $a_2 = 10$, $a_3 = 13$, d=3, putting the value in the formula of S_n we get
   $S_{12} = \frac{12}{2}[2 \times 7 + (12-1) \times 3] = 6[14 + 33] = 6 \times 47 = 282$]
7. The sum of all two-digit odd numbers is
   (a) 2575 (b) 2475 (c) 2524 (d) 2425
   Ans: (b) 2475 [All two-digit odd numbers are 11,13,15,... 99, which are in A.P.
   $\therefore$ Sum $= \frac{45}{2}[11 + 99] = 452 \times 110 = 45 \times 55 = 2475$]
8. If $(p+q)^{th}$ term of an A.P. is m and $(p-q)^{tn}$ term is n, then pth term is
   (a) mn (b) $\sqrt{mn}$ (c) $\frac{(m-n)}{2}$ (d) $\frac{(m+n)}{2}$
   Ans:(d) $\frac{(m+n)}{2}$ [$a_{p+q} = m$, $a_{p-q} = n \Rightarrow a + (p+q-1)d = m$...(i)$\Rightarrow a + (p-q-1)d = m$...(ii)
   On adding (i) and (ii), we get $2a + (2p-2)d = m + n \Rightarrow a_n = \frac{(m+n)}{2}$]
9. The number of multiples lie between n and n^2 which are divisible by n is
   (a) n + 1 (b) n (c) n – 1 (d) n – 2
   Ans: (d) n – 2 [Multiples of n from 1 to n^2 are $n \times 1, n \times 2, n \times 3, \ldots, n \times n$
   There are n numbers. Thus, the number of multiples of n which lie between n and n^2 is (n – 2) leaving first and last in the given list: Total numbers are (n – 2)]

3. Find whether -150 is a term of the A.P. 17, 12, 7, 2, ... ?
   Ans: -150 is not the term of given AP. [using formula of a_n, we get $n = \frac{172}{5}$
4. Which term of the progression 20, 192, 183, 17 ... is the first negative term?
   Ans: For first negative term, $a_n < 0$, $n > 27.5$ ∴ Its negative term is 28th term.
5. Find how many two-digit numbers are divisible by 6?
   Ans: AP is 12, 18, 24, ...,96 [using the formula, $a + (n-1)d = a_n \Rightarrow n = 15$.]
6. Find the middle term of the A.P. 6, 13, 20, ..., 216?
   Ans: using the formula $a_n = a + (n-1)d$, we get n=31, Middle term = $(\frac{n+1}{2})^{th}$ term. 16^{th} term, a_{16} =111
7. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero?
   Ans: Put the value in $S_n=0$, we get n=19
8. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 2 - 3n$.
   Ans: For n = 1, a_1 = -1and for n = 25, a_{25} = -73, using the formula $S_n = \frac{n}{2}[a_1 + a_n]$, we get $S_{25} = -925$.
9. The first and the last terms of an AP are 8 and 65 respectively. If the sum of all its terms is 730, find its common difference.
   Ans: Using $S_n = 730$, we get n=20, using the formula of a_n, we get d=3
10. A man receives Rs. 60 for the first week and Rs. 3 more each week than the preceding week. How much does he earn by the 20th week?
   Ans: AP is 60,63, 66, ..., using the formula $S_n = \frac{20}{2}[120 + 57]$ =1770.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. The angles of a triangle are in A.P., the least being half the greatest. Find the angles.
   Ans: Let the angles be a – d, a, a + d; a > 0, d > 0, a – d + a + a + d = 180°,
   On solving we get a=60°, d= 20°, the angles are 40°, 60° and 80°.
2. The 4^{th} term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11^{th}term.
   Ans: If $a_4 = 0$ then $a + 3d = 0 \Rightarrow a = -3d$. To prove: $a_{25} = 3 \times a_{11}$
   LHS =$a + 24d = -3d + 24d \Rightarrow 21d$ and RHS=$3(a + 10d) \Rightarrow 3(-3d + 10d) \Rightarrow 21d$
   From above, $a_{25} = 3(a_{11})$
3. The 7^{th} term of an A.P. is 20 and its 13^{th} term is 32. Find the A.P.
   Ans: On solving two equations which were framed as per the condition given, we get a=8 and d=2,
   so the AP is 8,10,12, ...
4. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where s, denotes the sum of its first n terms.
   Ans: On putting the values in $S_5 + S_7 = 167$, we get $12a + 31d = 167$ and on solving $S_{10} = 235$ we get $10a + 45d = 235$, after simplification we get a=1, d=5.so the A.P. is 1, 6, 11...
5. Which term of the A.P. 3, 14, 25, 36, ... will be 99 more than its 25th term?
   Ans: According to the Question, $a_n = 99 + a_{25} \Rightarrow a + (n-1)d = 99 + a + 24d$
   after simplification we get n=34.
6. The 14^{th} term of an AP is twice its 8^{th} term. If its 6^{th} term is -8, then find the sum of its first 20 terms.
   Ans: $a_{14} = 2a_8$ and $a_6 = -8$ [Given], after simplification we get a=1 and d=-1. Now $S_{20} = -340$
7. If the sum of first 7 terms of an A.P is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.
   Ans: Given: $S_7 = 49$, $S_{17} = 289$, on putting the values we get 2a+6d=14 and 2a+16d=34, after simplification we get a=1 and d=2 so $S_n = n^2$.
8. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.
   Ans: $S_n = 400$, using the formula $S_n = \frac{n}{2}[a_1 + a_n]$, we get n=16, Now, $a_n = 45$, on simplification d $= \frac{8}{3}$

9. Two AP's have the same common difference. The first term for one of these is -1, and that for the other is -8. Find the difference for their 4th term.
Ans: 1st A.P. with the first term -1 as well as the common difference d is $-1, -1+d, -1+2d, \ldots$
2nd A.P. with the first term -8 as well as the common difference d is $-8, -8+d, -8+2d, \ldots$
The 4th term of first A.P. is: $a_4 = -1+3d$,
The 4th term of second AP is: $A_4 = -8+3d$
Difference $a_4 - A_4 = (-1+3d)-(-8+3d) \Rightarrow 7$.
The difference between their 4th term is 7.

10. An A.P. contains 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find out the A.P.
Ans: As, n = 37 (odd), Middle term will be $\frac{n+1}{2}$ th term = 19th term
So, the three middle most terms will be, 18th, 19th and 20th terms.
As per the question, $a_{18} + a_{19} + a_{20} = 225. \Rightarrow a = 75 - 18d$.. (1),
the last three terms will be 35th, 36th and 37th terms. $a_{35} + a_{36} + a_{37} = 429$
$a + 35d = 143$.. (2),
On solving d=4 & a=3, so the AP is 3,7,11...

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.
Ans: We can see, that the given penalties are in the form of A.P. having $a = 200$ and $d = 50$
Penalty that has to be paid if contractor has delayed the work by 30 days = S_{30}
By the formula of sum of nth term, $S_{30} = \frac{30}{2}[2(200) + (30-1)50] = 27750$
Therefore, the contractor has to pay Rs 27750 as penalty.

2. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.
Ans: Let the cost of 1st prize be Rs. P, 2nd prize = Rs. P − 20 and cost of 3rd prize = Rs. P − 40
We can see that the cost of these prizes are in the form of A.P., having common difference as −20 and first term as P. Thus, a = P and d = −20. Given that, $S_7 = 700$
By the formula of sum of nth term, $7/2\,[2a + (7-1)d] = 700$. On solving a = 160
Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

3. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?
Ans: It can be observed that the number of trees planted by the students is in an AP.
1, 2, 3, 4, 5....................12 with $a = 1$ and $d = 2-1 = 1$,
by using the formula of S_n, $S_{12} = \frac{12}{2}[2(1)+(12-1)(1)] = 78$
Therefore, number of trees planted by 1 section of the classes = 78, Number of trees planted by 3 sections of the classes = 3×78 = 234. so, 234 trees will be planted.

4. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)

10. If a, b, c, d, e are in A.P., then the value of a – 4b + 6c – 4d + e is

(a) 0 (b) 1 (c) -1 (d) 2

Ans: (a) 0

[Let common difference of A.P. be x∴ b = a + x, c = a + 2x, d = a + 3x and e = a + 4x
Given equation a-4b + 6c-4d + e = a – 4(a + x) + 6(a + 2x) – 4(a + 3x) + (a + 4x)
= a – 4a – 4x + 6a + 12x – 4a – 12x + a + 4x = 8a – 8a + 16x – 16x = 0]

ASSERTION AND REASONING QUESTIONS

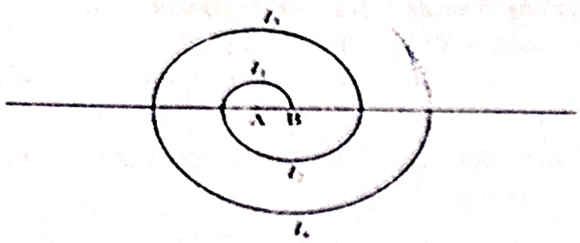
Directions: In the questions below, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct option.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

1. **Assertion(A):** Let the positive numbers a, b, c be in A.P., then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in A.P.
Reason (R): If each term of an A.P. is divided by abc, then the resulting sequence is also in A.P.
Ans: (a)
2. **Assertion(A):** If S_n is the sum of the first n terms of an AP, then its nth term an is given by $a_n = S_n - S_{n-1}$ **Reason (R):** The 10th term of the A.P. 5, 8, 11, 14, ... is 35.
Ans: (c)
3. **Assertion(A):** The sum of the series with the nth term, $a_n = (9 - 5n)$ is (465), when n = 15.
Reason (R): Given series is in A.P. and sum of n terms of an A.P. is $S_n = \frac{n}{2}[2a + (n-1)d]$
Ans: (d)
4. **Assertion (A):** The value of n is 18, if a = 10, d = 5, a_n = 95
Reason (R): The formula of general term an is $a_n = a+(n-1)d$.
Ans: (a)
5. **Assertion (A):** The 11th term of an AP 7,9,11, 13,, ,.......is 67
Reason (R): if S_n is the sum of first n terms of an AP then its nth term an is given by $a_n = S_n - S_{n-1}$
Ans: (d)
6. **Assertion (A):** The first term of an AP is m and the common difference is p then the 13[th] term is a+10p
Reason (R) : In an AP, $a_n = S_n - S_{n-1}$
Ans: (d)
7. **Assertion (A):** In an AP, $S_n = n^2 + n$ then $a_{20} = 40$
Reason (R): In an AP $d = a_n - a_{n-1}$
Ans: (b)
8. **Assertion (A):** If the nth term of an AP is $2n^2 - 1$, then the sum of the first n terms is n^3
Reason (R): If a, l and n are first, last and number of terms of an AP respectively, then $S_n = \frac{n}{2}(a + l)$
Ans: (d)
9. **Assertion (A):** The value of n, if a = 10, d = 5, a_n = 95.
Reason (R): The formula of general term an is $a_n = a+(n-1)d$.
Ans: (a)
10. **Assertion (A):** The nth term of an arithmetic progression is given by $a_n = a+(n-1)d$.
Reason (R): In an AP, the difference between any two consecutive terms is constant.
Ans: (a)

VERY SHORT ANS.WER TYPE QUESTIONS (2 MARKS QUESTIONS)

1. Find the common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$
Ans: The common difference = -1 [Using $d = a_2 - a_1$]
2. Find the 9th term from the end (towards the first term) of the A.P. 5, 9,13, ..., 185.
Ans: n^{th} term from the end = $l - (n-1)d$, 9^{th} term from the end = 185 – (9 – 1)4 = 153



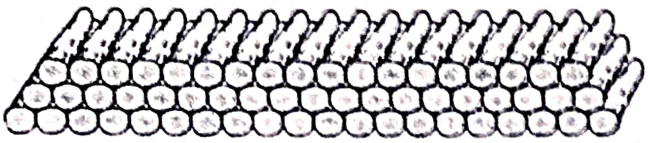
ANS Perimeter of a semi-circle $= \pi r$. *Therefore,* $p_1 = \frac{\pi}{2}$ cm, $p_2 = \pi$ cm, $p_3 = \frac{3\pi}{2}$ cm

where, p_1, p_2, p_3 are the lengths of the semi-circles. So the AP is $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$

With $a = \frac{\pi}{2}, d = \frac{\pi}{2}$, By the sum of n term formula, Sum of the length of 13 consecutive circles is:

$S_{13} = \frac{13}{2}[2(\frac{\pi}{2}) + (13-1)\frac{\pi}{2}] = 143$ cm

5. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Ans: We can see that the numbers of logs in rows are in the form of an A.P.20, 19, 18...*with* $a = 20$ and

$d = -1$. Let a total of 200 logs be placed in n rows. *Thus,* $S_n = 200$. By the sum of nth term formula,

$S_n = \frac{n}{2}[2(20)+(n-1)(-1)] \Rightarrow n^2 - 41n + 400 = 0$ on solving we get n= 16 or $n = 25$.

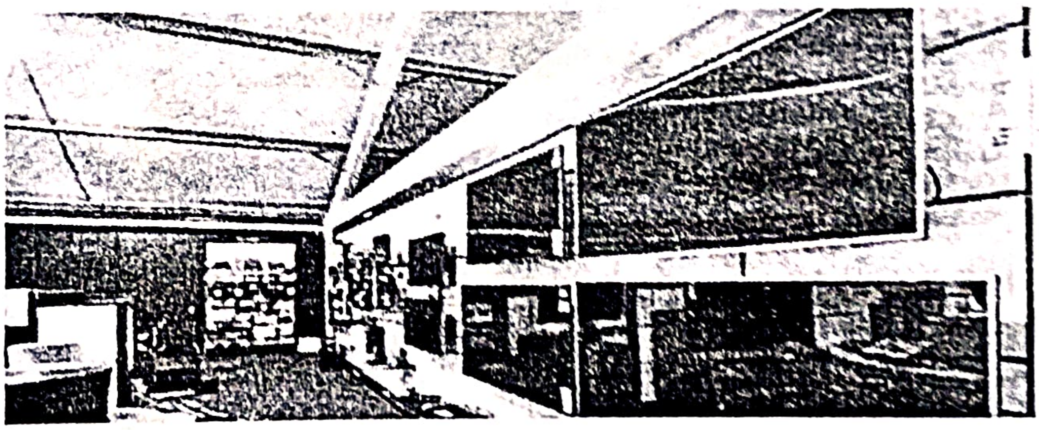
By the nth term formula, we get $a_{16} = 5$ and $a_{25} = -4$. It can be seen, the number of logs in 16th row is 5, as

the numbers cannot be negative.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

CASE BASED QUESTIOS (4 MARKS QUESTIONS)

1. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.



Based on the above information, Answer the following questions:

(i) Find the production during first year.

(ii) Find the production during 8th year.

(iii)Find the production during first 3 years.

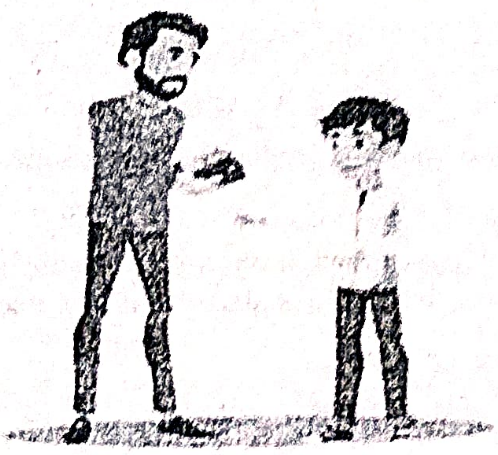
(iv)In which year, the production is Rs 29,200.

(v) Find the difference of the production during 7th year and 4th year.

Ans: (i).Rs 5000,

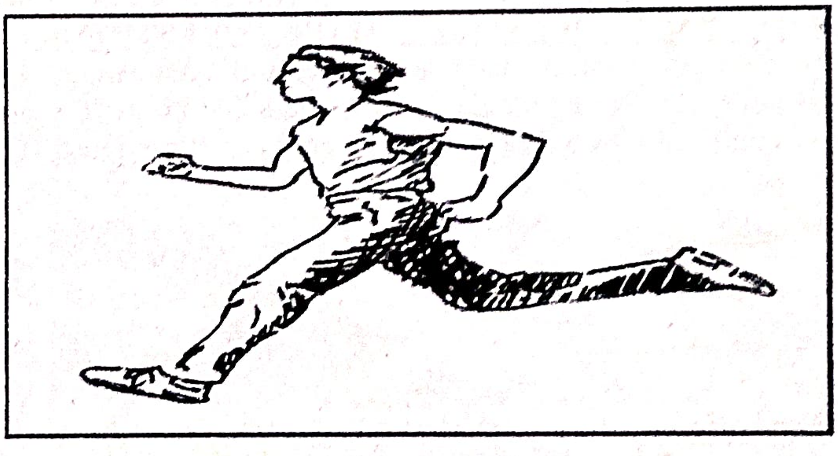
(ii). Production during 8th year is (a+7d) = 5000 + 7(2200) = 20400
(iii). Production during first 3 year = 5000 + 7200 + 9400 = 21600
(iv). n = 125
(v)Difference = 18200 - 11600 = 6600.

2. Anuj gets pocket money from his father everyday. Out of the pocket money, he saves Rs 2.75 on first day, Rs 3 on second day, Rs 3.25 on third day and so on.



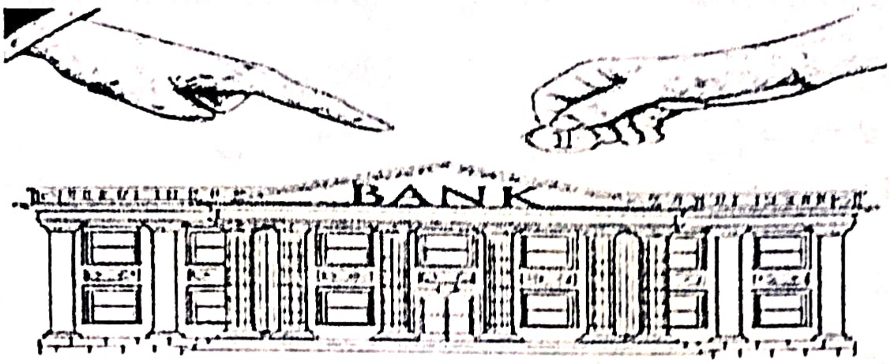
On the basis of above information, answer the following questions.
(i) What is the amount saved by Anuj on 14th day?
(ii) What is the total amount saved by Anuj in 8 days?
(iii) What is the amount saved by Anuj on 30th day?
(iv) What is the total amount saved by him in the month of June, if he starts savings from 1st June?
Ans: (i) Rs.6 (ii) Rs.29 (iii)Rs.10 (iv)Rs.191.25

3. Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.



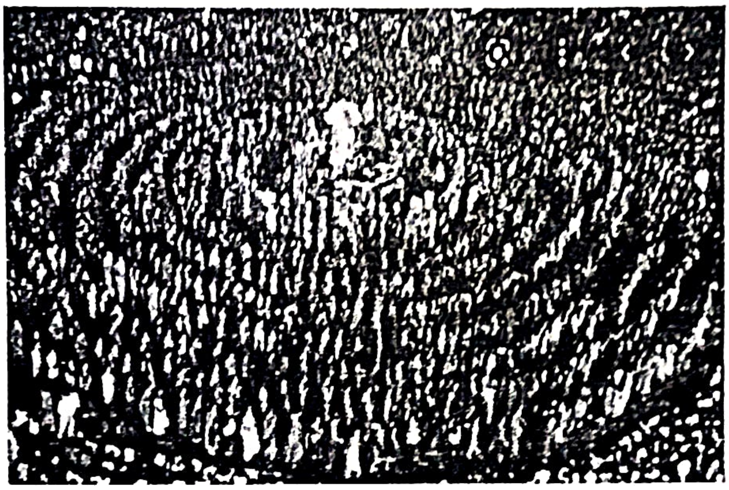
On the basis of the above answer the following questions
(i) Write an AP for the given situation.
(ii)What is the minimum number of days he needs to practice till his goal is achieved.
(iii) If nth term of an AP is given by a = 2n + 3 then find the common difference of an AP.
(iv) Find the value of x, for which 2x, x+ 10, 3x + 2 are three consecutive terms of an AP.
Ans: (i) 51, 49, 47...., (ii) 11, (iii) 2, (iv) 6

4. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, Answer the following:



(i) Find the amount paid by him in 30th installment.
(ii) Find the amount paid by him in the 30 installments.
(iii) What amount does he still have to pay after 30th installment?
(iv) If total installments are 40 then find amount paid in the last installment.
Ans: (i) 3900, (ii) 73500, (iii) 44500, (iv) 4900

5. Garba is a form of dance which originates from the state of Gujarat in India. The name is derived from the Sanskrit term Garbha. Many traditional garbas are performed around a centrally lit lamp or a picture or statue of the Goddess Shakti. Traditionally, it is performed in circles around this lamp during the nine-day Indian festival Navarātrī.



In the garba ground in a colony, it was decided to fix the participants to 1500. The dancers were to perform in concentric rings in such a way that each succeeding (outer) ring had 10 more participants than the previous(inner) ring.
(i) If the first ring had place for 30 participants, how many dancers will be there in the 10th row?
(ii) In how many circles will 1500 dancers dance as per this arrangement?
(iii) How many dancers will be left out after 12th row?
(iv) Suppose there are 13 circles of dancers, how many dancers are there in the middle ring?
Ans: (i) $a_{10} = 30 + 9 \times 10 = 120$ (ii) n= 15,
(iii) No of dancers left out after 12th row= 1500-1020= 480, (iv) $a_7 = 100$

HIGHER ORDER THINKING SKILLS

1. Find the '6th' term of the A.P.: $\frac{2m+1}{m}, \frac{2m-1}{m}, \frac{2m-3}{m}$

Ans: $a = \frac{2m+1}{m}$ $d = \frac{-2}{m}$, $a_6 = \frac{2m+1}{m} + 5 * \frac{-2}{m} \Rightarrow \frac{2m-9}{m}$

2. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean between 'a' and 'b', then, find the value of 'n'.

Ans: As per given condition

$\frac{a+b}{2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$(a^n + b^n)(a + b) = 2(a^{n+1} + b^{n+1}) \Rightarrow a^{n+1} + a^n b + b^n a + b^{n+1} = 2a^{n+1} + 2b^{n+1}$

$\Rightarrow a^{n+1} + b^{n+1} = a^n b + b^n a \Rightarrow (a^{n+1} - a^n b) - (b^n a - b^{n+1}) = 0 \Rightarrow a^n (a - b) - b^n (a - b) = 0 \Rightarrow (a^n - b^n)(a - b) = 0$

$\Rightarrow$ Either $(a^n - b^n) = 0$ or $(a - b) = 0$ But $a \neq b \Rightarrow a - b \neq 0$

$(a^n - b^n) = 0 \Rightarrow a^n = b^n \Rightarrow \left(\frac{a}{b}\right)^n = 1 \Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0 \Rightarrow n = 0.$

3. If p^{th} term of an A.P. is $\frac{1}{q}$ and qth term is $\frac{1}{p}$, then prove that the sum of the first 'pq' terms is ½(pq+1).

   Ans: p^{th} term $= a_n = a + (n - 1)\,d = \frac{1}{q}$ ....(1) q^{th} term $= a_n = a + (n - 1)\,d = \frac{1}{p}$ ....(2)

   Subtracting (2) from (1), we obtain $\Rightarrow d = \frac{1}{pq}$, $\Rightarrow a = \frac{1}{pq}$. By applying the formula

   $S_{pq} = \frac{pq}{2}\,[2a + (pq - 1)\,d]$

   $S_{pq} = \frac{pq}{2}\,[\frac{2}{pq} + (pq - 1)\frac{1}{pq}] = \frac{1}{2}(pq + 1)$

   Thus, the sum of the first pq terms of the A.P is $\frac{1}{2}(pq + 1)$

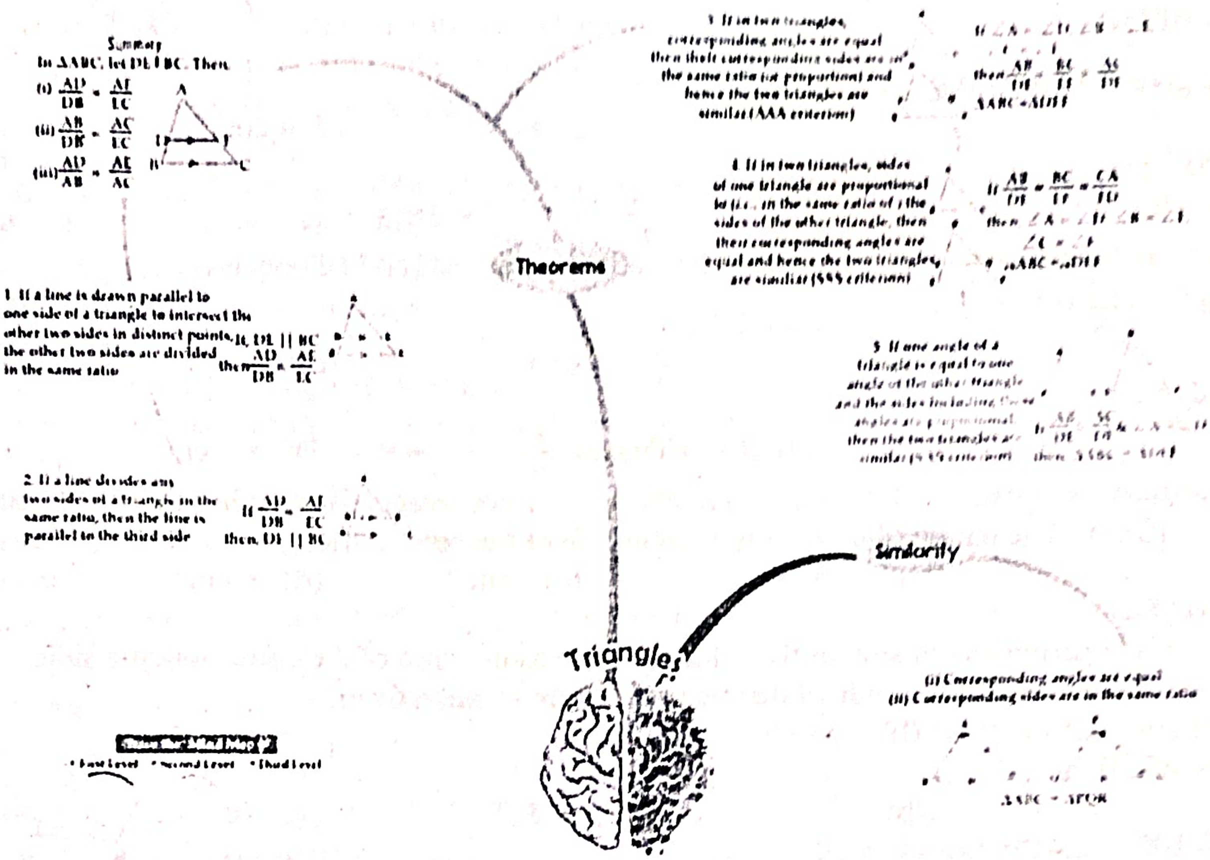
4. Solve the equation: $1 + 4 + 7 + 10 + ... + x = 287$

   Ans: Given, the <u>sum</u> of the terms up to x is 287. Here a=1, d=3.
   using the formula of S_n we get $3n^2 - n - 574 = 0$.
   On solving the QE we get n=14 and $-\frac{41}{3}$
   Since a negative term is not possible, $n = -\frac{41}{3}$ is neglected.
   Now we use the formula $a_n = a + (n - 1)d$.
   So, $x = 1 + (14 - 1)(3) \Rightarrow x = 40$.

5. Find three numbers in A.P. whose sum is 21 and their product is 231.

   Ans: Let the three consecutive terms be a – d, a, a + d,
   as per the condition: $a - d + a + a + d = 21 \Rightarrow 3a = 21 \Rightarrow a = 7$
   Given; $(7 - d)(7)(7 + d) = 231 \Rightarrow (7 - d)(7 + d) = 33$
   $\Rightarrow 49 - d^2 = 33 \Rightarrow d^2 = 16 \Rightarrow d = -4, 4$.
   The AP is 11, 7, 3 or 3, 7, 11.

CHAPTER- 6
TRIANGLES
MIND MAPPING



Concepts:

I. Criteria for similarity of Triangles

1. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)
2. If in two triangles, sides of one triangle are proportional to (in the same ratio of) the i.e., sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. (SSS criterion)
3. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)

II. Theorems:

Basic Proportionality Theorem (BPT) (Thales Theorem) Proof :

Video link: https://youtu.be/qdC3hs4hBsQ?si=eBS4sgLeL8MF5otp

Converse of BPT (Proof is not required)

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1. If in Δ ABC and Δ PQR, we have $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then:

(a) ΔPQR ~ΔCAB (b) ΔPQR ~ΔABC (c) ΔCBA ~ΔPQR (d) ΔBCA ~ΔPQR

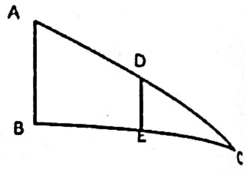
Ans. (a) ΔPQR ~ΔCAB

2. Which of the following is NOT a similarity criterion?

(a) AA (b) SAS (c) AAA (d) RHS

Ans. (d) RHS

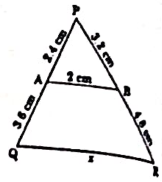
3. In ΔABC, DE || AB. If AB = a, DE = x, BE = b and EC = c, then x in terms of a,b and c.



(a) $\frac{ac}{b}$ (b) $\frac{ac}{b+C}$ (c) $\frac{ab}{c}$ (d) $\frac{ab}{b+c}$

Ans. (b) $\frac{ac}{b+C}$

4. In the given figure, if AB I QR, the value of x =

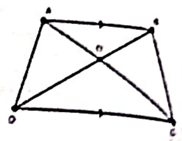


(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Ans. (c) 5 cm

Here, $\Delta PAB \sim \Delta PQR$, $\frac{QR}{AB} = \frac{PQ}{PA}, \frac{x}{2} = \frac{3.6+2.4}{2.4}$, $\frac{x}{2} = \frac{6}{2.4}$, $x = \frac{6 \times 2}{2.4} = 5$ cm.

5. In trapezium ABCD, AB I CD, AB = 3 cm. The diagonals AC and BD intersect at O such that $\frac{AO}{CO} = \frac{BO}{DO} = \frac{1}{2}$, then, CD = _____ cm.



(a) 3 (b) 4 (c) 5 (d) 6

Ans. (d) 6

Given : $\frac{AO}{CO} = \frac{BO}{DO} = \frac{1}{2}$, by SAS rule, $\Delta AOB \sim \Delta COD$. So, $\frac{AB}{DC} = \frac{1}{2}, \frac{3}{DC} = \frac{1}{2}$, DC = 6 cm.

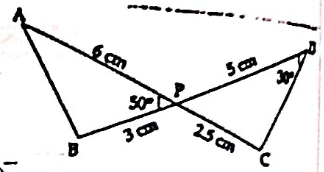
6. The perimeters of two similar triangles are 25cm and 15 cm respectively. One side of the first triangle is 10 cm. The length of the corresponding side of the second triangle is

(a) 4 cm (b) 5 cm (c) 6 cm (d) 10 cm

Ans. (c) 6 cm

The ratio of the perimeters of similar triangles are same as the ratio of the corresponding sides. So, 25:15 = 10: corresponding side of the second triangle ∴ side = 6 cm.

7. In the given figure, if ΔABP~ ΔDCP, the value of ∠B =

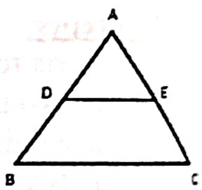


(a) 30^0 (b) 50^0 (c) 80^0 (d) 10

Ans. (d) 100^0 ∠APB =∠DPC = 50^0

Here ΔABP~ ΔDCP, ∠B = ∠C = $180^0 - (\angle DPC + \angle D) = 180^0 - (30^0 + 50^0) = 100^0$.

8. In the given figure DE||BC, if AD = 5.6cm, DB = 4cm and AE = 7cm then, what will be the value of AC?

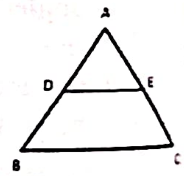


(a) 2.8cm (b) 5.6cm (c) 5 cm. (d) 12 cm.

Ans. (d) 12 cm

Here $\frac{AD}{DB} = \frac{AE}{EC}, \frac{5.6}{4} = \frac{7}{EC}$, $EC = \frac{4 \times 7}{5.6} = \frac{28}{5.6} = 5$ cm. So, AC = AE + EC = 7 + 5 = 12cm.

9. In the given figure if DE || BC, AD=3 cm, BD = 4cm and BC = 14cm, then DE equals to

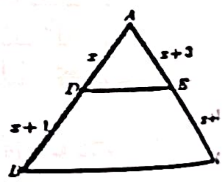


(a) 7cm (b) 6cm (c) 4cm (d) 3cm

Ans: (b) 6cm $\Delta ADE \sim \Delta ABC$ (by AAsimilarity)

∴ $\frac{DE}{BC} = \frac{AD}{AB}, \frac{DE}{14} = \frac{3}{AD+DB}$, $DE = \frac{3 \times 14}{3+4} = \frac{42}{7} = 6$ cm.

10. In ΔABC, if DE || BC, then the value of x is ________



(a) 1 (b) 2 (c) 3 (d) 4.

Ans: (c) 3

$\frac{AD}{DB} = \frac{AE}{EC}, \frac{x}{x+1} = \frac{x+3}{x+5}$, $x^2 + 4x + 3 = x^2 + 5x$, x = 3

ASSERTION AND REASONING BASED QUESTIONS

Directions: In the questions below, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct option.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

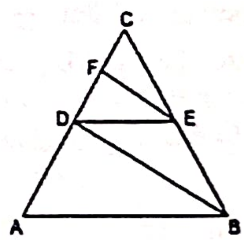
(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

1. **Assertion (A):** If two angles of one triangle are respectively equal to two angles of another triangle, then the triangles are similar.
Reason (R): If two angles of one triangle are equal to two angles of another triangle, then the third angle is also equal.
Ans: (a)
2. **Assertion (A):** Two right triangles are similar if the hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of another triangle.
Reason (R): AAA is the only criterion for similarity in right triangles.
Ans: (c)
3.**Assertion (A):** If a line is drawn parallel to one side of a triangle to intersect the other two sides, it divides the sides in the same ratio.
Reason (R): In any triangle, the line drawn from the midpoint of one side to the midpoint of another side is parallel to the third side.
Ans : (b)
4. **Assertion (A):** All congruent triangles are similar.
Reason (R): Congruent triangles have equal corresponding sides and equal corresponding angles.
Ans: (a)
5.**Assertion (A):** Two triangles are similar if their corresponding sides are in the same ratio.
Reason (R): This condition is known as the SSS criterion of similarity.
Ans:(a).
6.**Assertion (A):** In triangle ABC, if D and E are points on AB and AC respectively such that DE || BC, then ΔADE ~ ΔABC.
Reason (R): A line parallel to one side of a triangle divides the other two sides proportionally and corresponding angles are equal.
Ans: (a)
7. **Assertion (A):** The SSS criterion is sufficient to prove the similarity of two triangles.
Reason (R): All corresponding angles will be equal if the sides are proportional.
Ans: (a)
8. **Assertion (A):** If in triangles ABC and PQR, ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R, then the triangles are similar.
Reason (R): If all angles of one triangle are equal to all angles of another triangle, the corresponding sides are also equal.
Ans: (c)
9.**Assertion (A):** Two triangles having equal corresponding angles are always congruent.
Reason (R): All congruent triangles are similar.
Ans: (d)
10.**Assertion (A):** In and, if PQ/XY = QR/YZ = RP/ZX, then ΔPQR~ ΔXYZ
Reason (R): The SSS similarity criterion uses the equality of angles.
Ans: (c)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

1. In the given figure AB||DE, BD||EF. Prove that $DC^2=CF\times AC$.

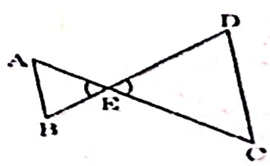


Ans. In ΔCAB, DE || AB, ∴ $\frac{DC}{AC}=\frac{CE}{BC}$ (By BPT)----(i)

In ΔCDB, EF|| BD, ∴ $\frac{CF}{DC}=\frac{CE}{BC}$ (By BPT)----(ii)

From (i) and (ii) $\frac{DC}{AC}=\frac{CF}{DC}$ ∴ $DC^2=CF\times AC$.

2. In the given figure, $\frac{EA}{EC}=\frac{EB}{ED}$, prove that $\Delta EAB{\sim}\Delta ECD$

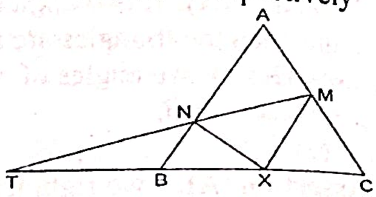


Ans. Given , $\frac{EA}{EC}=\frac{EB}{ED}$

$\angle AEB=\angle CED(Vertically\ opposite\ angles)$

∴ $\Delta EAB{\sim}\Delta ECD$(SAS similarity Criteria)

3. X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$.

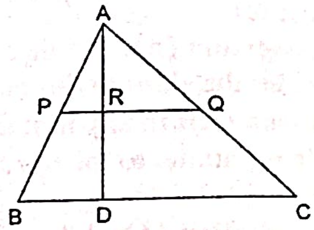


Ans. In ΔTXM, XM ||BN $\therefore \frac{TB}{TX} = \frac{TN}{TM}$ ......(1)

In ΔTMC, $\frac{TX}{TC} = \frac{TN}{TM}$ ....(2)

From 1 and 2, we have $\frac{TB}{TX} = \frac{TX}{TC}$ $\therefore TX^2 = TB \times TC$.

4. In the given figure, AP = 3 cm, AR = 4.5 cm, AQ = 6 cm, AB = 5 cm, AC = 10 cm. Find the length of AD



Ans: In ΔABC,

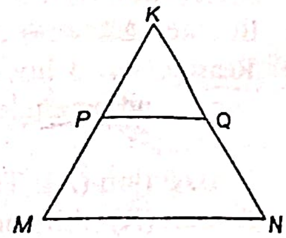
$$\frac{AP}{AB} = \frac{3}{5} \quad \text{....... (i)}$$

$$\frac{AQ}{AC} = \frac{6}{10} = \frac{3}{5} \quad \text{....... (ii)}$$

From (i) and (ii), we get $\frac{AP}{AB} = \frac{AQ}{AC} \Rightarrow PQ \parallel BC$

In ΔABD, PR || BD $\Rightarrow \frac{AP}{AB} = \frac{AR}{AD} \Rightarrow \frac{3}{5} = \frac{4.5}{AD} \Rightarrow AD = 7.5cm$

5.In Figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and KN = 20.4 cm. Find KQ.



Ans: In ΔKMN, we have PQ || MN

$\therefore \frac{KP}{PM} = \frac{KQ}{QN}$ [Basic proportionality Theorem]

$$\Rightarrow \frac{KP}{PM} = \frac{KQ}{KN - KQ} \Rightarrow \frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

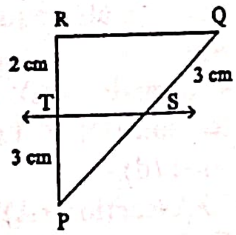
$\Rightarrow 4(20.4 - KQ) = 13KQ$

$\Rightarrow 81.6 - 4KQ = 13KQ$

$\Rightarrow 17KQ = 81.6$

$$KQ = \frac{81.6}{17} = 4.8cm$$

6. In the below figure, if ST || QR. Find PS.

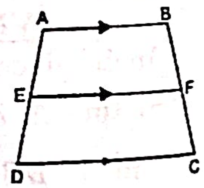


Ans. By Basic proportionality theorem,

$$\frac{PS}{QS} = \frac{PT}{RT}$$

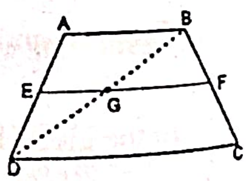
$$\Rightarrow \frac{PS}{3} = \frac{3}{2} \Rightarrow PS = \frac{9}{2}cm$$

7. In the given figure, if ABCD is a trapezium in which AB || DC || EF, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans. Given AB || DC || EF

Join BD intersecting EF at G.



In ΔDAB, EG || AB

$\frac{AE}{DE} = \frac{BG}{DG}$ (BPT) ....(i)

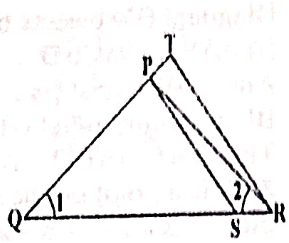
In ΔDBC, GF || DC

$\frac{BG}{GD} = \frac{BF}{FC}$ (BPT).............(ii)

From (i) & (ii)

$\frac{AE}{DE} = \frac{BF}{FC}$

8.In the figure $\frac{QR}{QS}=\frac{QT}{PR}$ and 1= 2. Show that ΔPQS ~ ΔTQR.



ANS. In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR (i)

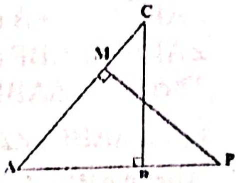
Given, $\frac{QR}{QS}=\frac{QT}{PR}$ ∴ $\frac{QR}{QS}=\frac{QT}{QP}$

In ΔPQS and ΔTQR, $\frac{QR}{QS}=\frac{QT}{QP}$ and ∠Q = ∠Q

∴ ΔPQS ~ ΔTQR [SAS similarity criterion]

9. In the given figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that: (i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA}=\frac{BC}{MP}$



Ans. In ΔABC and ΔAMP,

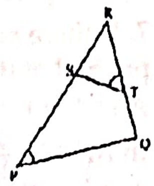
∠ABC = ∠AMP (Each 90°) ∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

∴ $\frac{CA}{PA}=\frac{BC}{MP}$ (Corresponding sides of similar triangles)

10. S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS.

Show that ΔRPQ ~ ΔRTS.



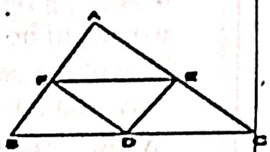
Ans. In ΔRPQ and ΔRST, ∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

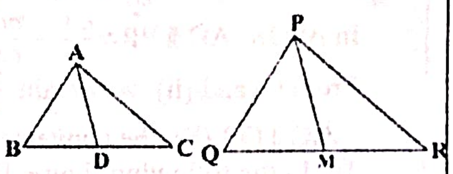
1. In ΔABC, D, E and F are midpoints of BC, CA and AB respectively. Prove that Δ *FBD* ~ Δ DEF and Δ DEF ~ Δ ABC



Ans. ∵ D, E, F are the mid points of BC, CA, AB respectively, EF|| BC, DF || AC, DE || AB

So, BDEF is a parallelogram, ∠DBF = ∠FED , ∠BDF = ∠EFD ∴ Δ *FBD* ~ Δ DEF and also, DCEF is parallelogram, ∴ Δ DEF ~ Δ ABC

2.Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR



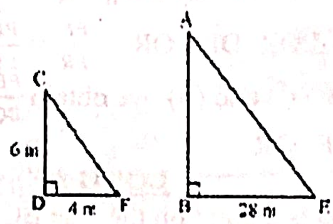
Ans. Given that , $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$ ∴ $\frac{AB}{PQ}=\frac{2BD}{2QM}=\frac{AD}{PM}$ ∴ $\frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$ --(1)

∴ ΔABD ~ ΔPQM ∴ ∠B = ∠ Q

Now in Δs ABC and PQR, $\frac{AB}{PQ}=\frac{BC}{QR}$ and ∠B = ∠ Q

∴ ΔABC ~ ΔPQR

3. A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.



Ans. Let AB and CD be a tower and a pole respectively. Let the shadow of BE and DF be the shadow of AB and CD respectively. At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, ∠DCF = ∠BAE And, ∠CDF = ∠ABE (Tower and pole are vertical to the ground)

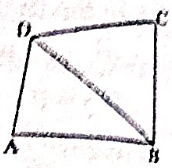
∴ ΔABE ~ ΔCDF (AAA similarity criterion)

$\frac{AB}{CD}=\frac{BE}{DF}$, $\frac{AB}{6}=\frac{28}{4}$ ∴ AB= 42m.

4. In the given figure, ABCD is a quadrilateral.

Diagonal BD bisects both ∠B and ∠D. Show that:

(i) ΔABD ~ ΔCBD (ii) AB=BC

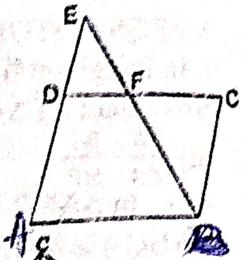


Ans. (i)BD bisects ∠B and ∠D, so: ∠ABD = ∠CBD and ∠ADB = ∠CDB

BD is common in both triangles.

Therefore, ΔABD ~ ΔCBD by ASA (Angle-Side-Angle)

5. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.



Ans.AB ∥ DC and AB = DC (opposite sides of a parallelogram)

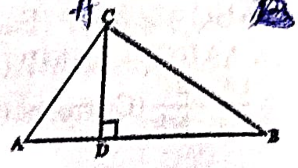
AD ∥ BC and AD = BC

∠ABE = ∠CFB (alternate interior angles)

∠AEB = ∠CBF (VOA) ∠ABE = ∠CFB ∠AEB = ∠CBF

Therefore, ΔABE ~ ΔCFB by AA similarity

6. In ΔABC, ∠ACB = 90⁰ and CD⊥ AB, Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.

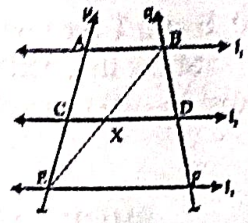


Ans. ΔABC~ΔCBD ∴ BC^2 = AB.BD ---(1)

ΔABC~ΔACD ∴ AC^2 = AB.AD ---(2)

Dividing(1) by (2) we get $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.

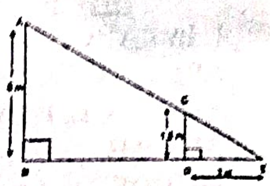
7. If three parallel lines are intersected by two transversals, then prove that the intercepts made by them on the transversals are proportional.



Ans. Let $l_1 \parallel l_2 \parallel l_3$, Join BE.

In ΔABE, $\frac{AC}{CE} = \frac{BX}{XE}$, In ΔBEF, $\frac{BX}{XE} \frac{BD}{DF}$ ∴ $\frac{AC}{CE} = \frac{BD}{DF}$.

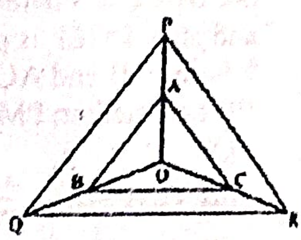
8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how far she is away from the base of the pole.



Ans. Let AB be the pole and CD the woman.

ΔABC~ΔCDE ∴ $\frac{AB}{CD} = \frac{BE}{DE}$ ∴ $\frac{6}{1.5} = \frac{3+BD}{3}$ ∴BD = 9 m.

9. In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB ∥ PQ and AC ∥ PR. Show that BC ∥ QR.



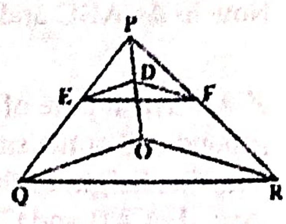
Ans. In ΔPOQ, AB ∥ PQ ∴ $\frac{OA}{AP} = \frac{OB}{BQ}$(Basic proportionality theorem) --- (i)

In ΔPOR, AC ∥ PR∴ $\frac{OA}{AP} = \frac{OC}{CR}$ = (Basic proportionality theorem) ---(ii)

From (i) and (ii), we obtain $\frac{OB}{BQ} = \frac{OC}{CR}$

∴BC ∥ QR (By the converse of basic proportionality theorem

10. In the following figure, DE ∥ OQ and DF ∥ OR, show that EF ∥ QR.



Ans. In ΔPOQ, DE ∥ OQ ∴ $\frac{PE}{EQ} = \frac{PD}{DO}$ (basic proportionality theorem) (i)

In ΔPOR, DF ∥ OR ∴ $\frac{PF}{FR} = \frac{PD}{DO}$ (basic proportionality theorem) (ii)

From (i) and (ii), we obtain $\frac{PE}{EQ} = \frac{PF}{FR}$

∴EF ∥QR.

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$, prove that $CA^2 = CB.CD$

Ans. In Δ ABC, D is a point on side BC such that ∠ADC = ∠BAC

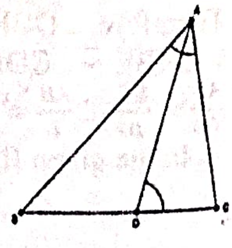
In Δ CBA and Δ CDA

∠C = ∠C (common)

∠ BAC = ∠ADC (given)

∴ Δ CBA ~ Δ CAD (By AA similarity)

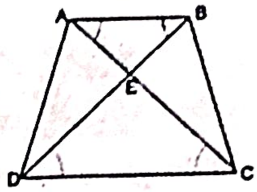
∴their corresponding sides are proportional



∴ $\frac{CB}{CA} = \frac{CA}{CD}$ ∴CA^2 = CB . CD

2.State and Prove Thales theorem.
Ans:

3. (i)Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
(ii)Using the above theorem, prove that if ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point E $\frac{AE}{BE} = \frac{CE}{DE}$.

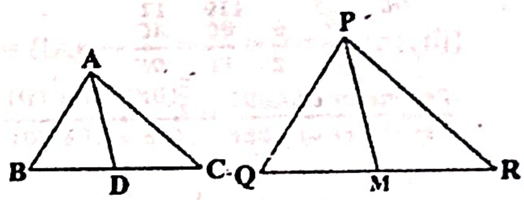


Ans. (i) BPT---- same as the answer of question 2.
(ii) ABCD is a trapezium with AB || DC. diagonals intersect each other at point E.
Draw EO||AB
Draw EO ||AB
In Δ ABD EO ||AB, hence $\frac{AO}{DO} = \frac{BE}{DE}$ (BPT)........(i)
InΔ ACD EO || DC, Hence $\frac{AO}{DO} = \frac{AE}{CE}$ (BPT)..........(ii)
From (i) and (ii) $\frac{AE}{BE} = \frac{CE}{DE}$.

4. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Ans:

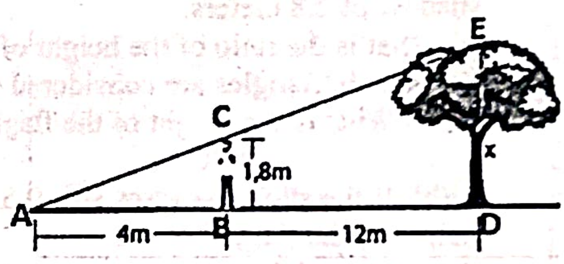
5. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.



Ans. ΔABC ~ ΔPQR. ∴ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ... (1)
Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)
Since AD and PM are medians, BD= $\frac{BC}{2}$ and QM= $\frac{QR}{2}$ ... (3)
From (1) and (3), $\frac{AB}{PQ} = \frac{BD}{QM}$.
In ΔABD and ΔPQM, ∠B = ∠Q AND $\frac{AB}{PQ} = \frac{BD}{QM}$...
∴ ΔABD ~ ΔPQM (By SAS similarity criterion ∴ $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$.

CASE BASED QUESTIONS(4 MARKS QUESTIONS)

1. Sharon went to a park with her father. From a point A where Sharon was standing, her father and tip of a tree come in a straight line as shown in the figure. She remembers lesson on similar triangles. She asked her father to stay in that place and tried to estimate the height of the tree. Based on this information, answer the following questions.



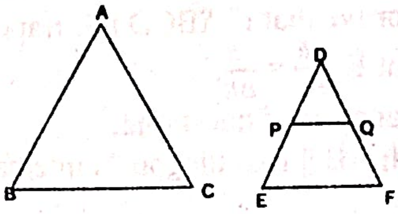
(i) By what criterion are the triangles ΔABC and ΔADE similar?
(ii) Write the ratios of corresponding sides of the triangles in the above question.
(iii)(a) Find the height of the tree.
(OR)
(iii)(b) If a 6 feet flag staff forms a shadow of 15 feet on the ground, find the height of the tree which forms a shadow of 60 feet at the same time.
A. i) AA similarity criterion (as ∠A = ∠A & ∠ABC = ∠ADE)
(ii) $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

(iii) (a) $\frac{AB}{AD}=\frac{BC}{DE}$ ∴ $\frac{AB}{AB+BD}=\frac{BC}{DE}$ ∴ $\frac{4}{16}=\frac{1.8}{DE}$ ∴$DE = 4 \times 1.8 = 7.2m$.

(iii) (b) $\frac{Height\ of\ flag}{Shadow\ length}=\frac{Height\ of\ tree}{Shadow\ length}$, $\frac{6}{15}=\frac{Height\ of\ tree}{60}$

∴Height of tree $= \frac{360}{15} = 24$ m

2.



Triangle is a very popular shape used in interior designing. The picture given above shows a cabinet designed by a famous interior designer.

Here the largest triangle is represented by Δ ABC and smallest one with shelf is represented by Δ DEF. PQ is parallel to EF.

(i) Show that Δ DPQ ~ Δ DEF.

(ii) If DP= 50 cm and PE = 70 cm then find $\frac{PQ}{EF}$

(iii) (a) If 2AB = 5DE and Δ ABC ~ Δ DEF then show that the ratio of perimeters of Δ ABC to that of Δ DEF is 1:1.

(iii) (b) If AM and DN are medians of triangles ABC and DEF respectively then prove that Δ ABM ~ Δ DEN.

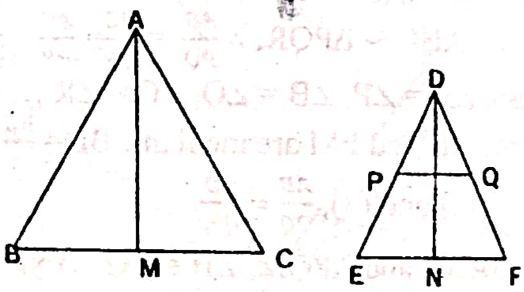
Ans. (i)∠DPQ = ∠DEF ∠PDQ =∠EDF Therefore Δ DPQ ~ Δ DEF

(ii)DE = 50 + 70 = 120 cm

$\frac{DP}{DE}=\frac{PQ}{EF} \Rightarrow \frac{50}{120}=\frac{5}{12}$

(iii) (a) $\frac{AB}{DE}=\frac{5}{2}=\frac{BC}{EF}=\frac{AC}{DF} \longrightarrow AB=\frac{5}{2}DE$

$\frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta DEF}=\frac{\frac{5}{2}(DE+EF+FD)}{(DE+EF+FD)}=\frac{5}{2}$



(iii) (b) $\frac{AB}{DE}=\frac{BC}{EF}=\frac{\frac{1}{2}BC}{\frac{1}{2}EF}=\frac{BM}{EN}$

Also ∠B =∠E Therefore Δ ABM ~ Δ DEN.

3. A student sees a flagpole and wants to find its height. He uses a 1-meter stick placed vertically on the ground. At a certain time of day, the stick casts a shadow of 0.4 meters and the flagpole casts a shadow of 2.8 meters.

(i) What is the ratio of the height of the stick to its shadow?

(ii) Which triangles are considered similar here?

iii(a). What is the height of the flagpole?

(OR)

iii(b). If the stick's shadow was 0.5 m and the pole's shadow was 3.5 m, what would be the new height of the flagpole?

Ans : (i) 1 : 0.4

(ii) Triangle formed by the stick and its shadow, and the triangle formed by the flagpole and its shadow.

(iii) (a) Using similar triangles:

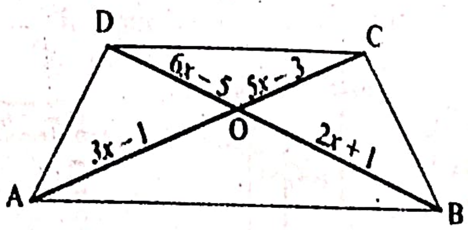
$\frac{1}{0.4}=\frac{h}{2.8} \Rightarrow h=\frac{1\times 2.8}{0.4}=7$ meters

(OR)

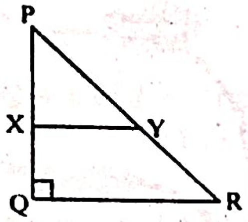
(iii) b. $\frac{1}{0.5}=\frac{h}{3.5} \Rightarrow h=\frac{1\times 3.5}{0.5}=7$ meters.

HIGHER ORDER THINKING SKILLS

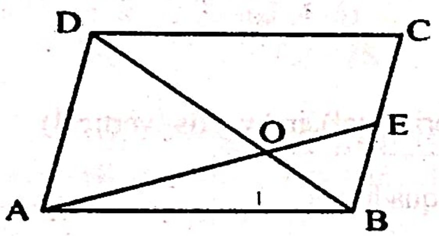
1. In the given figure, AB || DC and diagonals AC and BD intersect at O. If OA = 3x - 1 and OB = 2x + 1, OC = 2x + 1 and OD = 6x – 5, find the value of x.



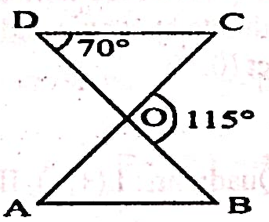
2. In the given figure, PQR is a right triangle with $\angle Q = 90^0$. If XY || QR, PQ = 6cm, PY = 4cm and PX:XQ = 1:2, then find the lengths of PR and QR.



3. In Δ PQR, MN || QR, using BPT, prove that $\frac{PM}{PQ} = \frac{PN}{PR}$.
4. In the figure given, ABCD is a parallelogram. AE divides BD in the ratio, 1:2. If BE = 1.5 cm. Find BC.



5. In the figure, Δ ODC ~ Δ OBA. $\angle BOC = 115^0$ and ∠~~BOC~~ ODC $= 70^0$.
Find, ∠DOC, ∠DCO, ∠OAB and ∠OBA.



ANSWERS:

1. x = 2 ,
2. PR = 10 cm, QR = 8 cm
3. PM/PQ = PN/PR
4. BC = 4.5 cm
5. ∠DOC = 65°, ∠DCO = 50°, ∠OAB = 65°, ∠OBA = 50°